Science of the Physical Universe 27

Science and Cooking:
From Haute Cuisine to Soft Matter Science

Final Exam Review Sheet

December 12, 2010

The goal of this sheet is to outline what is expected of you on the final exam.

The final exam will be quite similar to midterm, and this review sheet is organized in a similar fashion. As always, the learning objectives for the class are organized in categories for each week.

The goal is for you to be able to apply these learning objectives to understanding and thinking about real recipes – this is what you did already so successfully in the final project!

The learning objectives are listed first for each week. Additional problems for each learning objective are given at the end of this review sheet.

The questions on the exam will be very similar to these problems and will closely follow the learning objectives.

We have also listed on this review sheet the readings from the syllabus. The exam will likely also contain a set of true/false or identification questions covering major points from these readings.

You will be allowed to bring and use a single piece of 8.5 x 11 inch paper, with anything written on it that you want, to use in the exam.

We will also provide an equation sheet and a unit conversion sheet. This will be posted beforehand and will contain everything you need with equations and units for the exam. You are also allowed to wear your aprons!

Additional Problems

Foams and Emulsions

1. Milk is an example of an emulsified liquid. What are the continuous and dispersed phases of milk?
   Water is the continuous phase and milkfat is the dispersed phase.

2. Define hydrophobicity in one to two sentences. What kinds of (food) materials are almost always hydrophobic?
   Hydrophobicity is the quality of a compound or substance that will not mix with water. Lipids (fats & oils) never mix with water without help and so are hydrophobic.

3. The essential difference between milk and cream is the quantity of fat. Describe why cream is more viscous than milk. What would happen if you had a dairy product with super-high milkfat content?
   Having more and more of the dispersed phase reduces the ability of the continuous phase to flow, increasing the solution's viscosity. With enough fat dispersed into the milk, you could eventually halt the flow, creating a solid milk-based mayonnaise.

4. Draw what milk looks like under the microscope. Draw what this super-high fat solid milk emulsion might look like under a microscope. What shape are the dispersed bubbles in each drawing?
   For regular milk, the images should show something that has relatively few fat bubbles in a big bath of continuous water. The bubbles should be perfect spheres. In the solid drawing, the dispersed should be all pressed together such that those perfect spheres become smushed together and distorted in shape.

5. How is the dispersed phase in milk stabilized?
   Water and milkfat are allowed to mix because of surfactants. In milk, phospholipids and proteins surround the hydrophobic fat globules, allowing them to dissolve in the continuous phase.

6. In his lecture, Nandu Jubany prepared aioli with garlic, water, and olive oil. If he were to mix water and olive oil in the same way, would you expect it to form a stable emulsion? Why or why not?
   No, the garlic contains a small amount of surfactant, which stabilizes the emulsion.

7. Nandu used about a liter of olive oil in his aioli. If the oil droplets had a diameter of 50 microns, how many oil droplets did he form?
   The volume of each oil droplet is $4/3\ \pi\ R^3 = 6.4 \times 10^{-14}\ m^3$
   The total volume is $10^{-3}\ m^3$
   The total volume divided by oil droplet volume equals about 15 billion droplets (15×10^9 droplets).

8. Suppose you wanted to coat these droplets with lecithin. How much lecithin is required?
   The area of a lecithin molecule is 10^{-17} m^2
   The area of each bubble is $4 \pi R^2 = 7.8 \times 10^{-9}$ m^2
   The total area of the bubbles is the area per bubble times number of bubbles. This is equal to 117 m^2
   The total area divided by area per lecithin molecule equals the number of lecithin molecules required: 1.2×10^{19} molecules.

9. Instead of buying a pressure cooker, you wonder if you can achieve the same effect by using the pressure inside a droplet. In a water-in-olive-oil emulsion, how small do the water droplets need to be to reach an internal pressure of 15 psi above atmospheric (similar to the pressure in a pressure cooker)[1]? Use a surface energy of 23 mN/m.
   Start with the equation of the week: $\Delta p = 2\sigma/R$
   Solve for R: $R = 2\sigma/\Delta p$
   Substitute in the value: $R = (2)(23 \times 10^{-3}$ N/m$)/(100 \times 10^{3}$ Pa$) = 4.6\ 10^{-7}$ m

10. *Whipped cream from a can*: In an aerosol dispenser, the pressure of nitrous oxide (and hence the pressure difference across the bubbles) is 5 to 10 times atmospheric pressure (see "Dairy Fats and Technologies" ed by A Y Tamime).
    a. Express "5 times atmospheric pressure" in Pascals.
       $(5)(100 \times 10^{3}$ Pa$) = 5 \times 10^{5}$ Pa
    b. Assuming there is gas in the form of air bubbles, with a surface tension of 70 mN/m, what is the size of the bubbles?
       Start with the equation of the week: $\Delta p = 2\sigma/R$
       Solve for R: $R = 2\sigma/\Delta p$
       Substitute in the value: $R = (2)(70 \times 10^{-3}$ N/m$)/(5 \times 10^{5}$ Pa$) = 2.8 \times 10^{-7}$ m
    c. Manufacturers typically add mono- and di-glycerides to lower the surface tension, so it is closer to 30 mN/m. In that case, what is the size of the bubbles.
       Substitute in the value: $R = (2)(30 \times 10^{-3}$ N/m$)/(5 \times 10^{5}$ Pa$) = 1.2 \times 10^{-7}$ m
    d. If you created the bubbles by just stirring the whipped cream by hand, do you think the bubbles would be larger or smaller? Why?
       The bubbles would likely be larger, since the bubbles in the aerosol can start as dissolved gas, whereas whipping starts with large pockets of air incorporated into the liquid.

11. Describe in two to three sentences why one needs to whisk vigorously to create many foams and emulsions. Refer to an equation of the week.
   To create any emulsion one needs to create a lot of surface area, which requires the input of large amounts of energy according the equation: U-surface = sigma x

[1] Note: 1 psi = "pound per square inch" = 6894 Pa. Both are units of pressure. 1 atm (1 "atmosphere") = 101,325 Pa. This implies that 14.7psi = 1 atm. Sadly there are way too many units that people use for pressure! Remember though that if you want to compute the bubble radius in meters, you should use pressure in Pa, and surface energy in N/m.

surface area. The whisking provides the energy associated with the surface area needed for smaller and smaller drops.

Protein Folding and Unfolding

1. Given that an egg yolk solidifies at 63° C, deduce the percentage of hydrophobic residues in an egg yolk protein. For this problem, assume that electrostatic interactions are not important—ie there is no charge on the egg yolk protein. (Hints: The "percentage of hydrophobic residues" is the number of hydrophobic monomers / the total number of monomers or bonds. The energy of the hydrophobic interaction is about 10 kJ/mol. Use the equation of the week for week 5, neglecting electrostatic interactions).
   The equation of the week is: U(hydro) = N k_BT
   The hydrophobic energy is: U(hydro) = N(hydrophobic) U(bond)
   Solve for the fraction of hydrophobic amino acids: N(hydrophobic)/N
   This is equal to k_BT/U(bond)
   Substitute in values: (4.6 x 10^{-21} J)*(6.022 x 10^{23}/mol))/(10 x 10^3 J/mol)
   The fraction of hydrophobic residues is: 0.277 ~ 28%

2. In lecture, Michael mentioned that to make tofu you can curdle soy milk by heating it AND adding salt. A similar thing can happen when you add soy milk to coffee. But, coffee is not salty! What works alongside the heat of the coffee to make unfortunate clumps of "tofu" in a bad cup of coffee?
   The lower pH of the coffee denatures the proteins and leads to coagulation.

3. After lab, you have been wiping down the lab benches with bleach spray to kill any bacteria that may have found their way onto the surface. In 2008, scientists at the University of Michigan discovered that bleach kills bacteria by denaturating their proteins! The pH of bleach is 12.6; could it possibly be denaturing the proteins even though it is not acidic?
   Yes, highly alkaline environments also change the electrostatic interactions causing the proteins to denature.

4. *Briefly*, make a sketch of milk under the microscope, and also a sketch of curdled milk. Explain the difference between the two pictures using the equation of the week for Week 5. See the homework solutions for week 5.

Gelation

1. Describe the physical process of spherification. Describe the physical process of inverse spherification. What is the net charge on the ions involved?
   In spherification, a gelling agent (e.g. calcium ions) in an aqueous bath penetrates into a liquid droplet containing alginate, causing the droplet to solidify from the outside inwards. In inverse spherification, the alginate is in the aqueous bath outside of the liquid droplet and calcium is inside the droplet. Thus, the shell forms outwards from the liquid-liquid interface. The advantage of reverse spherification is that when the droplet is removed, the gelation reaction is stopped-all of the alginate that has gel is stuck to the droplet. In contrast, in normal spherification, extra calcium has diffused into the droplet and so the gelation reaction continues. Hence, reverse spherification gives greater control over the alginate shell thickness. The net charge of the ions must be at least +2 for the ions to serve as cross links for the negatively charged alginate strands.

2. How does this time change if you make your bubble tea boba using reverse spherification?
   The time is identical using normal and reverse spherification. The only difference between the two processes is whether the alginate is on the inside or the outside of the droplet.

3. You want to make your own bubble tea boba by spherifying chai tea into balls of diameter 2 cm. How long should you leave the spheres in the bath for the balls to spherify completely?
   Use the equation of the week: $l = \sqrt{(\pi D t)}$
   Solve for t: $t = l^2 / \pi D$
   Substitute in the values: $t = (1 \times 10^{-2}\ m)^2/((\pi)(1.5 \times 10^{-9}\ m^2/s)) = 2.1 \times 10^5\ s$
   This is roughly 6 hours

4. The diffusion constant of calcium decreases linearly with increasing viscosity of the liquid that the calcium ions must diffuse through. Suppose you want to spherify a thick chocolate syrup with a viscosity that is 100 times that of

water. The chocolate syrup has calcium ions in it, so this is a reverse spherification. Compared to the previous example, how much longer do you have to let the droplet sit in an alginate bath to achieve the same alginate layer thickness?

The reaction will take 100 times longer.

5. Ferran Adria's olive oil caviar, consisting of spherified droplets of olive oil, hits the American market and is an enormous hit. One of your friends calls you up and asks what the thickness of the gel layer is encapsulating the olive oil. Explain to him how to measure the thickness of the layer.

   Use the method you used in lab. First weigh the droplet; then calculate the radius of the droplet, assuming the density is the same as that of water ($1 g/cm^3$). Then pop the droplet and dry out the olive oil, so that you are just left with the shell. Weigh the shell. It weighs $4*pi*R^2*$ thickness * (density), where again the density is essentially the same as water. Using this you can get a measurement for the shell thickness.

Transglutaminase and Browning Reactions

1. *Heat resistant chocolate*: Gelatin (8% w/w) and transglutaminse (3%) are added to molten chocolate.
    a. If the resulting mixture cools to a solid with an elastic modulus of 10^7 Pa, what is length l of the cross-linking bonds?
    Start with the equation of the week: $E = k_BT/l^3$
    Solve for l: $l = (k_BT/E)^{1/3}$
    Substitute in the values: $l = ((4.14 \times 10^{-21}\ J)/(10^7\ Pa))^{1/3} = 7.5 \times 10^{-10}$ m
    b. When heated to 75 °C, the chocolate becomes less elastic, and the modulus drops to 10^4 Pa. Now what is the length of the cross-linking bonds?
    Substitute in the values: $l = ((4.14 \times 10^{-21}\ J)/(10^4\ Pa))^{1/3} = 7.5 \times 10^{-9}$ m
    c. Are the bonds between the gelatin strands and proteins in the cocoa solid due to covalent bonds, physical entanglement, or a mixture of these? Why?
    The transglutaminase creates covalent bonds, but the decrease in elasticity with the rise in temperature suggests that physical entanglement is also present.

2. *Vegetarian Meat Glue*: You are interested in creating a vegetarian meat glue. For efficient binding, should you combine transglutaminase with
    a. sodium alginate
    b. gellan
    c. soy protein – This is the only example containing the proper amino acids.
    d. agar agar

3. *Parmesan Noodles:* A noodle has an elastic modulus of about 10 kPa You want to make a shrimp (or parmesan, peanut butter or whatever your favorite protein is) noodle with transglutaminase (TG). How many cross-links per cubic centimeter does your TG need to make to give the same elasticity as a regular noodle?

Use the equation of the week $E = k_BT/l^3$
Solve for l: $l = (k_BT /E)^{1/3}$ = 7.39 nm
Need spacing of 7.39 nm in all three dimensions
The number of crosslinks (N) in 1 cm^3 = $1cm^3/(7.39\ nm)^3$
This equals 2.5×10^{18} cross links per cubic centimeter

4. *Microwave cookie:* You can "bake" a cookie in about a minute by heating the batter in a microwaveable cup. However, the surface will have a much paler appearance that cookies from the oven: why?

   The temperature of the batter doesn't exceed 100°C, so browning reactions do not occur.

Soil and Microbes

1. *Effect of Local Agriculture on Human Population*: Around 8,000 B.C., the knowledge about how to produce crops within human communities spurred local agriculture and had a huge effect on increasing human population by providing a means to maintain a consistent food supply, and thus decreasing the doubling time for human growth. At this time (~8,000 B.C.) the human population was approximately 5 million people. Before local agriculture, the doubling time for humans was about 100,000 years. After the local agriculture was developed, this doubling time decreasing this doubling time to about 2,000 years.

   a. Starting from an initial population of 5 million, what is the population after 1,000 years assuming pre-agricultural growth rate?
   Doubling time for pre-agricultural growth is ~100,000 years. Thus the population will be 5 million* $2^{(1000 \text{ years}/100000 \text{ year})}$= 5 million * $2^{(1/100)}$=5,034,800
   b. What is the population after 1,000 years using the growth rate associated with local agriculture, again with an initial population of 5 million?
   Now the doubling time is 2000 years.
   So the population will be 5 million*$2^{(1000 \text{ years}/2000 \text{ year})}$= 5 million * $2^{1/2}$
   This equals 7071068 = 7071000
   c. How many years would it take to reach the population you found in part (b) using the pre-agricultural growth rate?
   We need to find out how long it will take for the population to double,.This will take ~50,000 years:
   $(5 \times 10^6)(2^{t/100,000 \text{ years}}) = 7071000$
   $t = (10^5 \text{ years})*\log_2(7071000/5000000) = 50,000$ years
   alternative solution:
   $t = (10^5 \text{ years})*\log(7071000/5000000)/\log(2) = 50,000$ years
   d. Taking the starting population in 8,000 B.C. to be 5 million, if the local agriculture revolution had not taken place, what date would the time

you calculated in (c) correspond to? Has this date already passed, or is it in the future?

This date would be 42,000 A.D. This date is far into the future!

2. *Cultivating Optimal Microbial Composition in Compost*: Chef Dan Barber spoke about his technique of heating his compost at 142F so that the population of "bad" microbes that negatively influence plant growth is diminished while the "good" microbes remain. One possible reason that such an enrichment of the microbial community occurs is due to different half-life values, the amount of time the population is reduced by half, between the "bad" and "good" microbes at this temperature. Assume that in one gram of soil there are 1 billion "bad" microbes and 1 billion "good" microbes and the half-life of the "bad" microbes is 4 minutes while for the "good" microbes is 10 minutes at 142F.

   a. If we take a gram of soil and heat it to 142F, how long does it take to reduce the population of "bad" microbes down to 1 single microbe? At 142F, the bad microbes have a half life of 10 minutes.. Thus, we need to find the time for which 1 billion $/2^{(time/4minutes)}$ = 1, or 1 billion=$2^{(time/4minutes)}$. Taking the logarithm of both sides we have

      time = $\log_2$(1,000,000,000)*(4 minutes) ~ 120 minutes.

   b. How many "good" microbes remain when held at 142F for the amount of time found in part (a)?

      At 120 minutes, the number of good microbes is 1,000,000/$2^{(120/10)}$= 244,140.625=200,000 microbes

3. *Recolonizing your Gut Microbial Community*: There are approximately 100 trillion bacteria in your gut; the total mass of this bacteria equals approximately 2 kg. These bacteria are extremely beneficial and without them we would probably not survive. If you take antibiotics, such as penicillin (which is compound derived from another microbe, Penicillium fungi), it not only kills the microbes responsible for the infection you have but also these

necessary beneficial bacteria. It can take up to 4-8 weeks for these beneficial bacteria to re-establish in your gut if you take penicillin.

a. Assume that a typical dosage of penicillin kills the population of beneficial bacteria down to 100 billion. If it takes 6 weeks to grow this population back up to 100 trillion, what is the approximate doubling time of these beneficial bacteria?

   Start with the equation of the week: $N(t) = N_0 \exp(kt)$

   Solve for k: $k = \ln(N(t)/N_0)/t$

   Substitute the values for t = 6 weeks: $k = \ln(10^{14}/10^{11})/(6*7)$

   This sets $k = 0.164\ \text{day}^{-1}$

   To find the doubling time, use the fact that $k = \ln(2)/\tau$

   Solve for $\tau = \ln(2)/k = 4.2$ days

b. Yogurt contains live cultures of the bacteria necessary to keep your gut healthy. In fact, in 1 mL of yogurt there are approximately 1 billion live bacteria. A typical serving of yogurt is 8 ounces or ~240 mL. If you eat a serving of yogurt to help replenish your gut bacteria after taking a dosage of penicillin, how long does it now take for your gut to return to its original 100 trillion bacteria?

   Start with the equation of the week: $N(t) = N_0 \exp(kt)$

   Solve for t: $t = \ln(N(t)/N_0)/k = \ln(10^{14}/(3.4 \times 10^{11}))/(0.164\ 1/\text{day}) = 35$ days

c. Instead of waiting weeks for the bacteria to repopulate your gut, you want to eat enough yogurt to return your gut to a healthy 100 trillion bacteria and need to calculate the volume of yogurt you need to consume. How many 8 ounce servings of yogurt would you have to eat?

   In one serving of yogurt, there are (240 ml) (10^9/ml) = 240 billion bacteria

   The number of servings is $10^{14}/(2.4 \times 10^{11}) = 417$ servings

Desserts!

1. *Pies*: The book Cookwise by Shirley Corriher lists several suggestions for modifications to pie crust recipes. In one brief sentence, please explain why each of the following ingredient suggestions works:

   For tenderness:
   - Use low-protein flour. Less protein leads to fewer Maillard reactions.
   - Use some acidic ingredients. Lower pH inhibits Maillard reactions.

   For more color:
   - Use ingredients that container protein or sugar, like higher-protein flour, diary products, or sugars. The sugars and proteins leads to more Maillard reactions.
   - Use corn syrup. The higher fraction of sugar monomers leads to more Maillard reactions.
   - Add a little more baking soda (alkali). Higher pH promotes Maillard reactions.

2. *Mousse*: A simple mousse-like dessert can be made by mixing water and chocolate together in the proper ratio while heating over a double boiler, followed by whisking in an ice-cold bowl (see http://www.fooducation.org/2010/06/fool-proof-chocolate-chantilly-part-33.html). The batter must be mixed vigorously to incorporate gas (in this case, air) bubbles into the mousse. What are two methods used to accomplish this in other baking recipes?
   Yeast and chemical leavening (e.g. baking soda and vinegar).

Analysis of a Recipe: White bread!

This problem asks you to use essentially all the concepts of the course to analyze a single recipe.

Here is a recipe for a white bread:

2 (.25 ounce) packages active dry yeast
3 tablespoons [36 grams] white sugar
2 1/2 cups warm water (110 °F/45 °C)
3 tablespoons lard [36 grams], softened
1 tablespoon [12 grams] salt
6 1/2 cups [900 g] bread flour

Directions
In a large bowl, dissolve yeast and sugar in warm water. Stir in lard, salt and two cups of the flour. Stir in the remaining flour, 1/2 cup at a time, beating well after each addition. When the dough has pulled together, turn it out onto a lightly floured surface and knead until smooth and elastic, about 8 minutes.

Lightly oil a large bowl, place the dough in the bowl and turn to coat with oil. Cover with a damp cloth and let rise in a warm place until doubled in volume, about 1 hour.

Deflate the dough and turn it out onto a lightly floured surface. Divide the dough into two equal pieces and form into loaves. Place the loaves into two lightly greased 9x5 inch loaf pans. Cover the loaves with a damp cloth and let rise until doubled in volume, about 40 minutes.

Preheat oven to 425 °F (220 °C).

Bake at 375 °F (190 °C) for about 30 minutes or until the top is golden brown and the bottom of the loaf sounds hollow when tapped.

QUESTIONS

1. During baking, much of the water content of the bread is lost. What types of bonds are broken when water evaporates? What is the typical energy of those bonds? [Week 1, Phases of Matter]

   We use the equation of the week from week 1: $U = 3/2\ k_B T$, with T the transition temperature. Water boils at T = 100C = 373 K. Thus the typical energy of the bonds is

   $U = 3/2 * (1.38 \times 10^{-23}\ J/K) * (373\ K)$

   $U = 7.72 \times 10^{-21}\ J.$

   We often express this in kJ/mol:

   $U = 7.72 \times 10^{-21}\ J * (6.022 \times 10^{23}\ 1/mol)$

   U = 4.65 kJ/mol

   This corresponds to the energy of a hydrogen bond!

2. Each quarter-cup serving of bread flour contains 22g of carbohydrates and 4g of proteins. Taking into account all of ingredients in the bread [you can neglect the yeast!], what is the total number of calories in a loaf of bread? [Week 2, Food Components]

   Use the "4-4-9" rule for sugars, proteins, and fats, respectively.

   The total number of calories is:

   Carbs: (36g + 4*6.5*22g) * (4 Cal/g)

   Protein: (4*6.5*4g) * (4 Cal/g)

   Fat: (36 g) * (9 Cal/g)

   This equals: 3172 Calories (3000 Cal with sig figs)

3. Tired of working so hard to knead the dough, you give yourself a break by using a heavy book to press the dough. Suppose the book weighs 5 lb. You notice that if you shape the dough into a cube 15 cm to a side and put the book on it, the dough compresses to half its original height2. What is the elastic modulus of the bread dough? [Week 3, elasticity—recall the lab!]

   Use the equation of the week: $E = (F/A_0)/(\Delta L/L_0)$

[2] Assume that the height changes by a factor of 2 but the other sides of the cube keep the same length. Probably not realistic but it will do for here!

Convert the weight to kg: (5 lb)*(0.453 kg/lb) = 2.25 kg

Substitute in the value: $E = ((2.25\ kg)(9.8\ m/s^2)/(15^2\ cm^2))/(1/2)$ ~2 kPa

4. Suppose your loaf pan is 9″ x 5″ x 5″. What temperature does the center of the loaf reach? [Week 4: Heat transfer]

   Use the equation of the week: $T(t) = (T_{init} – T_{ext})\exp(-t/\tau)+T_{ext}$

   Use L = (5″/2)(2.54 cm/in) = 6.35 cm

   Calculate τ from $(1/\pi)(L^2/D) = (1/\pi)(6.35\ cm)^2/(1.4 \times 10^{-3}\ cm^2/s) = 9.2\ 10^3$ s

   Substitute in the values: T(1800 s) = (190 °C – 20 °C)exp(-1800/9200) + 190

   This yields T(1800 s) = 30.5 °C

5. Describe the phase transitions that turn the bread from a sticky dough into a fluffy, solid loaf. [Week 4: Heat transfer; Week 10, gluten discussion]

   The water turns form a liquid to a gas, leaving a rigid network of gluten protein and gelatinized starch.

6. How would you expect the bread to change if you use all-purpose flour instead of bread flour? (Hint: bread flour has an unusually high gluten content.) Describe how you would design an experiment to test the effect of gluten content of the flour on the bread properties. [Week 5; week 10]

   All-purpose four has a gluten content of about 11% by weight.

   Bread flour has a gluten content of 12 to 13% by weight.

   As an example of an experiment:

   - Prepare five batches of the dough with: gluten-free (e.g. rice) flour, cake flour, all-purpose flour, bread flour, and bread flour with vital wheat gluten added.
   - Look at the following properties of the final bread: total mass, range of pore size, elasticity.

7. Bread as a foam:
   i. Explain how bread is an example of a foam:

      Bread contains a dense packing of air pockets, stabilized by a solid network of proteins and starches.

   ii. Kneading the dough is necessary to create small air bubbles. If these initial air bubbles are about 10^{-5}m in diameter, and the surface tension of the dough with air is about 40 mN/m, what is the pressure inside these bubbles?

      Use the equation of the week: $\Delta p = 2\sigma/R$

      Substitute the values: $\Delta p = (2)(40 \times 10^{-3}\ \text{N/m})/(10^{-5}/2\ \text{m}) = 16\ \text{kPa}$

   iii. As the bread rises and bakes, the bubbles grow larger, until they are approximately 10^{-4}m in diameter. Now what is the pressure inside? [Week 6: Foams and Emulsions]

      Substitute the new values: $\Delta p = (2)(40 \times 10^{-3}\ \text{N/m})/(10^{-5}/2\ \text{m}) = 1.6\ \text{kPa}$

8. Bread as a gel:
   i. Explain how bread is an example of gelation.

      A sufficient number cross-links form between the gluten proteins, so that it turns from a viscous liquid into an elastic solid.

   ii. From your calculation of elasticity in problem 3, calculate the distance between cross-links in the gluten network. (If you did not get an answer to problem 4, estimate a elastic modulus; if you clearly state your estimate and solve the problem correctly you will still get full credit for this problem.) [Week 7: Gelation]

      Assume an elastic modulus of 2 kPa.

      Use the equation of the week: $E = k_BT/l^3$

      Solve for l: $l = (k_BT/E)^{1/3}$

      Substitute the values: $l = ((2000\ \text{Pa})/(4.14 \times 10^{21}\ \text{J}))^{1/3} = 1.3 \times 10^{-8}$ m

9. Complex phase changes in bread:
   i. Discuss the phase change that occurs on the surface of the bread. [Week 8: Complex Phase Change]

      The surface of the bread undergoes various browning reactions, between the sugars and proteins in the dough.

   ii. This phase change relies on heat diffusing into the dough to raise the temperature. Calculate an order of magnitude estimate for how thick you expect the crust to be. [Week 8 & Week 4]

      Use the heat diffusion equation: $L = \sqrt{(\pi D t)}$

      Substitute values: $L = \sqrt{(\pi)}(1.4 \times 10^{-3}\ \text{m}^2/\text{s})(60\ \text{s/min})(10\ \text{min}) = 1.5$ cm = order of 1cm

   iii. Suppose you are unhappy with your bread and want a darker crust. Describe at least two ways you might change the recipe in order to achieve this. [Week 8]

      A higher temperature would cause the browning reactions to occur faster. Raising the pH with baking soda or lye would also promote these reactions.

10. The doubling time for yeast is about 1.5 hours. How many yeast cells are in the dough after it has risen for a total of 1 hour 40 minutes? (Hint: to find the initial number of yeast cells, assume that each yeast cell has a mass of 10^{-10} grams; 1 ounce of yeast = 28 grams) [Week 9: Microbes]

Find the number of yeast cells: (0.5 oz)(28 g/oz)(1 yeast cell/10^{-10}g) = 1.4×10^{11} cells

The time constant $k = \ln(2)/\tau = 7.7 \times 10^{-3}$ min^{-1}

Use the equation of the week: $N(t) = N_0 \exp(k\ t)$

Substitute the values: N(100 min) = $(1.4 \times 10^{11}) \exp(7.7 \times 10^{-3}$ min^{-1} 100 min)

The number of yeast cells is: 3×10^{11} cells (this makes sense, since roughly one doubling time has elapsed)